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Rare kaon decays

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Abstract

We review some recent theoretical results on rare kaon decays. Particular attention is devoted to establish the short distance (direct CP violating) contribution to $K_L \to \pi^0 e\overline{e}$. This is achieved by a careful study of the long distance part. As byproduct, we discuss interesting chiral tests.

1 Introduction

Rare Kaon decays are going to play undoubtedly a crucial role to test the Standard Model (SM) and its extensions [1]. $K_L \to \pi^0 \nu\overline{\nu}$ and $K^\pm \to \pi^\pm \nu\overline{\nu}$ have the advantage not to be affected by long distance uncertainties and thus they are definetely very appealing [2, 3]; however due to the difficult detection they represent a real experimental challenge. There are also other interesting channels that can be studied either as byproduct of the previous ones or also as an independent search, like $K_L \to \pi^0 e\overline{e}$ or $K_L \to \mu\overline{\mu}$. Here the long distance contributions is in general not negligible and must be carefully studied in order to pin down the short distance part. The advantage is that these channels are more accessible experimentally.

2 $K \to \pi\nu\overline{\nu}$

The basic formalism to study systematically low energy physics is the OPE (Operator Product Expansion), where the physical processes are determined by an effective hamiltonian, written as a product of local operators O_i and (Wilson) coefficients c_i : $\mathcal{H}_{eff} = \sum_i c_i(\mu)$ $O_i(\mu)$; indeed the scale dependence must cancel in the product since physical processes are $\mu-$independent. For $K \to \pi\nu\overline{\nu}$ this writes as [2]

$$\mathcal{H}_{eff} = \frac{G_F}{\sqrt{2}} \frac{2\alpha}{\pi \sin^2 \Theta_W} \sum_{l=e,\mu,\tau} \left(\lambda_c X^l_{NL} + \lambda_t X(x_t)\right) \overline{s}_L \gamma^\mu d\,_L \overline{\nu}_L \gamma_\mu \nu_L + H.c. \qquad (1)$$

where $\lambda_q = V^*_{qs} V_{qd}$ and $X(x_t)$, X^l_{NL} the box+Z-penguin top and charm loop contributions. The latter is affected by strong radiative corrections. $SU(2)$ isospin symmetry relates hadronic matrix elements for $K \to \pi\nu\overline{\nu}$ to $K \to \pi l\overline{\nu}$. QCD corrections have been evaluated at next-to-leading order and the main uncertainties in (1) is due to the strong corrections to X^l_{NL}, that implies 5% error on the determination of λ_t from $K^{\pm} \to \pi^{\pm}\nu\overline{\nu}$.

The structure in (1) leads to a pure CP violating contribution to $K_L \to \pi^0\nu\overline{\nu}$, induced only from the top loop contribution and thus proportional to $\Im m(\lambda_t)$ and free of hadronic uncertainties. This leads to the prediction [2]

$$B(K_L \to \pi^0\nu\overline{\nu})_{SM} = 4.25 \times 10^{-10} \left[\frac{\overline{m}_t(m_t)}{170GeV}\right]^{2.3} \left[\frac{\Im m(\lambda_t)}{\lambda^5}\right]^2 . \tag{2}$$

On the other hand $K^{\pm} \to \pi^{\pm}\nu\overline{\nu}$ receives CP conserving and CP violating contributions proportional respectively to $\Re e(\lambda_c)X^l_{NL} + \Re e(\lambda_t)X(x_t)$ and $\Im m(\lambda_t)X(x_t)$. If one takes into account the various indirect limits on CKM elements one obtains [1, 2]

$$B(K_L \to \pi^0\nu\overline{\nu})_{SM} \ = (2.8 \pm 1.1) \times 10^{-11} \tag{3}$$

$$B(K^{\pm} \to \pi^{\pm}\nu\overline{\nu})_{SM} = (0.79 \pm 0.31) \times 10^{-10} \tag{4}$$

To be compared respectively with the experimental results [4] and [5]

$$B(K_L \to \pi^0\nu\overline{\nu}) \ \leq 5.9 \times 10^{-7} \qquad B(K^{\pm} \to \pi^{\pm}\nu\overline{\nu}) = \left(4.2^{+9.7}_{-3.5}\right) \times 10^{-10} \tag{5}$$

Lately it has been pointed out the possibility of new physics to substantially enhance the SM predictions ($\sim$a factor 10 in the branching) [6] through effects that could be parametrized by an effective $\overline{s}dZ$ vertex Z_{ds}; the CP violating contribution $\Im m(Z_{ds})$ and consequentely $K_L \to \pi^0\nu\overline{\nu}$ is constrained by the value of ε'/ε, while $\Re e(Z_{ds})$ and $K^{\pm} \to \pi^{\pm}\nu\overline{\nu}$ are limited by $K_L \to \mu\overline{\mu}$. Indeed it has been shown that it is possible to evaluate in a realiable way the long distance contributions to $K_L \to \mu\overline{\mu}$ and thus to constrain the short distance part [7].

The recent value of ε'/ε [8], though not incompatible with the SM, allows large values for new sources of CP violating contributions.

3 $K_L \to \pi^0 e^+ e^-$

3.1 Direct CP violating contributions

The electromagnetic interactions allow new structures to this decay compared to $K \to \pi\nu\overline{\nu}$. The effective Hamiltonian for $s \to d\ell^+\ell^-$ transitions is known at next–to–leading order and for $\mu < m_c$ reads [9]

$$\mathcal{H}^{|\Delta S|=1}_{eff} = \frac{G_F}{\sqrt{2}} V^*_{us}V_{ud}\Big[\sum_{i=1}^{6,7V}(z_i(\mu) + \tau y_i(\mu))Q_i(\mu) \ + \ \tau y_{7A}(M_W)Q_{7A}(M_W)\Big] + \text{h.c.}, \tag{6}$$

where $\tau = -(V^*_{ts}V_{td})/(V^*_{us}V_{ud})$, $Q_{1,..,6}$ are the usual four-quarks operators and

$$Q_{7V} \ = \ \overline{s}\gamma^{\mu}(1-\gamma_5)d\,\overline{\ell}\gamma_{\mu}\ell \qquad \text{and} \qquad Q_{7A} \ = \ \overline{s}\gamma^{\mu}(1-\gamma_5)d\,\overline{\ell}\gamma_{\mu}\gamma_5\ell \tag{7}$$

are generated by electroweak penguins and box diagrams.

Thus there is i) a direct CP violating contribution analogous to (1) (additional single photon exchange contributions are smaller), ii) indirect CP –violating contribution $K_L = K_2 + \tilde{\varepsilon} K_1 \stackrel{\tilde{\varepsilon} K_1}{\rightarrow} \pi^0 \stackrel{*}{\gamma} \rightarrow \pi^0 e^+ e^-$ and iii) a $CP-$ conserving contribution: $K_L \rightarrow \pi^0 \stackrel{*}{\gamma}\stackrel{*}{\gamma} \rightarrow \pi^0 e^+ e^-$.

The prediction for the direct CP violation contribution is [9]

$$B(K_L \rightarrow \pi^0 e^+ e^-)^{SM}_{CPV-dir} = 0.69 \times 10^{-10} \left[\frac{\bar{m}_t(m_t)}{170 GeV}\right]^2 \left[\frac{\Im m(\lambda_t)}{\lambda^5}\right]^2 ;$$

using the present constrains on $\Im m(\lambda_t)$ one obtains [1]

$$B(K_L \rightarrow \pi^0 e^+ e^-)^{SM}_{CPV-dir} = (4.6 \pm 1.8) \times 10^{-12}.$$

3.2 Indirect CP violating contribution, $K_S \rightarrow \pi^0 e^+ e^-$ and $K^\pm \rightarrow \pi^\pm l^+ l^-$

The interplay between long and short distance contributions to $K \rightarrow \pi \stackrel{*}{\gamma}$ is manifest in (6), where the Wilson coefficient of the Q_7-operator is scale-dependent while the Q_7-matrix element is not. Thus the scale-dependence must be cancelled by the four-quark operators in (6), which have large long distance (non-perturbative) contributions.

Chiral Symmetry is the appropriate framework to evaluate these contributions. Chiral pertubation theory (χPT) [10] is an effective field theory based on the following two assumptions: i) the pseudoscalar mesons are the Goldstone bosons (G.B.) of the symmetry $SU(3)_L \otimes SU(3)_R$ spontaneously broken to $SU(3)_V$, ii) there is a *(chiral) power counting* , i.e. the theory has a small expansion parameter: $p^2/\ \Lambda^2_{\chi SB}$ and/or $m^2/\ \Lambda^2_{\chi SB}$, where p is the external momenta, m the masses of the G.B.'s and $\Lambda_{\chi SB}$ is the chiral symmetry breaking scale: $\Lambda_{\chi SB} \sim 4\pi F_\pi \sim 1.2\ GeV$. Being an effective field theory, loops and counterterms are required by unitarity and have to be evaluated order by order.

$K \rightarrow \pi \stackrel{*}{\gamma}$ ($K^\pm \rightarrow \pi^\pm \stackrel{*}{\gamma}$ and $K_S \rightarrow \pi^0 \stackrel{*}{\gamma}$) decays start at $\mathcal{O}(p^4)$ in χPT with loops (dominated by the $\pi\pi-$cut) and counterterm contributions [11]. Higher order contributions ($\mathcal{O}(p^6)$) might be large, but are not completely under control since new (and with unknown coefficients) counterterm structures appear [12]. In Ref. [13] we have parameterized the $K \rightarrow \pi \stackrel{*}{\gamma} (q)$ form factor as

$$W_i(z) = G_F M_K^2 \,(a_i + b_i z) + W_i^{\pi\pi}(z) , \qquad\qquad i = \pm, S \tag{8}$$

with $z = q^2/M_K^2$, and where $W_i^{\pi\pi}(z)$ is the loop contribution, given by the $K \rightarrow \pi\pi\pi$ unitarity cut and completely known up to $\mathcal{O}(p^6)$. All our results in that reference were given in terms of the unknown parameters a_i and b_i, expected of $\mathcal{O}(1)$. At the first non-trivial order, $\mathcal{O}(p^4)$, $b_i = 0$, while a_i receive counterterm contributions not determined yet. At $\mathcal{O}(p^6)$, $b_i \neq 0$, while a_i receive new counterterm contributions. Due to the generality of (8), we expect that $W_i(z)$ is a good approximation to the complete form factor of $\mathcal{O}(p^6)$.

Experimentally the $K^+ \rightarrow \pi^+ l^+ l^-$ (l=e, μ) widths and $K^+ \rightarrow \pi^+ e^+ e^-$ slope have been measured, while in the muon channel the slope has been measured only after our paper [14].

We can fix a_+ and b_+ from the $K^+ \to \pi^+ e^+ e^-$ rate and spectrum respectively. Thus we can predict the ratio (R) of the widths μ/e; which however overestimates the experimental findings [13, 15] by 2.2 σ 's. Due to the generality of the form factor (8) we thought that the experimental situation should improve. Indeed data from E865 [14] in $K^+ \to \pi^+ \mu^+ \mu^-$ confirms our prediction: i) it is possible to describe well both leptonic channels with (8) and $B(K^+ \to \pi^+ \mu^+ \mu^-)$ is now 3.2 σ 's larger than the previous measurement [15] and even more interestingly ii) the fit with (8), i.e. with the genuine chiral contributions $W_i^{\pi\pi}(z)$, is better ($\chi^2\ /d.o.f. \sim 19.9/9$) than just a linear slope ($\chi^2/d.o.f. \sim \chi^2_{\min} + 9$), showing the validity of the chiral expansion.

There is no model independent relation among a_S and a_+ and thus a secure determination of $B(K_L \to \pi^0 e^+ e^-)_{CP-indirect}$ requires a direct measurement of $B(K_S \to \pi^0 e^+ e^-)$, possibly to be performed at KLOE at DAΦ NE [13]. The dependence from b_S is very mild and thus we predict $B(K_S \to \pi^0 e^+ e^-) \simeq 5.2\, a_S^2 \times 10^{-9}$.

If we include the interference term among direct and indirect the CP –violating terms we obtain

$$B(K_L \to \pi^0 e^+ e^-)_{CPV} = \left[15.3\, a_S^2 - 6.8 \frac{\Im m \lambda_t}{10^{-4}}\, a_S + 2.8 \left(\frac{\Im m \lambda_t}{10^{-4}} \right)^2 \right] \times 10^{-12} . \quad (9)$$

A very interesting scenario emerges for $a_S \lesssim -0.5$ or $a_S \gtrsim 1.0$. Since $\Im m \lambda_t$ is expected to be $\sim 10^{-4}$, one would have $B(K_L \to \pi^0 e^+ e^-)_{CPV} \gtrsim 10^{-11}$ in this case. Moreover, the $K_S \to \pi^0 e^+ e^-$ branching ratio would be large enough to allow a direct determination of $|a_S|$. Thus, from the interference term in (9) one could perform an independent measurement of $\Im m \lambda_t$, with a precision increasing with the value of $|a_S|$.

3.3 CP conserving contributions: "$\gamma\gamma$" intermediate state contributions

The general amplitude for $K_L(p) \to \pi^0 \gamma(q_1) \gamma(q_2)$ can be written in terms of two independent Lorentz and gauge invariant amplitudes $A(z,y)$ and $B(z,y)$:

$$\begin{aligned} M^{\mu\nu} = & \frac{A(z,y)}{m_K^2} (q_2^\mu q_1^\nu - q_1 \cdot q_2 g^{\mu\nu}) + \frac{2\, B(z,y)}{m_K^4} (-p \cdot q_1\, p \cdot q_2\, g^{\mu\nu} - q_1 \cdot q_2\, p^\mu p^\nu \\ & + p \cdot q_1\, q_2^\mu p^\nu + p \cdot q_2\, p^\mu q_1^\nu) \end{aligned} \quad (10)$$

where $y = p \cdot (q_1 - q_2)/m_K^2$ and $z = (q_1 + q_2)^2/m_K^2$. Then the double differential rate is given by

$$\frac{\partial^2 \Gamma}{\partial y\, \partial z} = \frac{m_K}{2^9 \pi^3} [\, z^2 \,|\, A + B\,|^2 + \left(y^2 - \frac{\lambda(1, r_\pi^2, z)}{4} \right)^2 |\, B\,|^2\,] , \quad (11)$$

where $\lambda(a,b,c)$ is the usual kinematical function and $r_\pi = m_\pi/m_K$. Thus in the region of small z (collinear photons) the B amplitude is dominant and can be determined separately from the A amplitude. This feature is crucial in order to disentangle the CP-conserving contribution $K_L \to \pi^0 e^+ e^-$.

We must warn however about the danger of the potentially large background contribution from $K_L \to e^+ e^- \gamma\gamma$ to $K_L \to \pi^0 e^+ e^-$ [16].

The two photons in the A-type amplitude are in a state of total angular momentum $J = 0$ (J, total diphoton angular momentum), and it turns out that for this contribution $A(K_L \to \pi^0 e^+ e^-)_{J=0} \sim m_e$ (m_e electron mass) [17]; however the higher angular momentum state B-type amplitude in (10), though chirally and kinematically suppressed for $A(K_L \to \pi^0\gamma\gamma)$, generate $A(K_L \to \pi^0 e^+ e^-)_{J\neq 0}$ competitive with the CP violating contributions [12].

The leading finite $\mathcal{O}(p^4)$ amplitudes of $K_L \to \pi^0\gamma\gamma$ generates only the A–type amplitude in Eq. (11). This underestimates the observed branching ratio, $(1.68 \pm 0.07 \pm 0.08) \times 10^{-6}$ [18] by a large factor but reproduces the experimental spectrum, predicting no events at small z. The two presumably large $\mathcal{O}(p^6)$ contributions have been studied: i) the $\mathcal{O}(p^6)$ unitarity corrections [19, 20, 21] that enhance the $\mathcal{O}(p^4)$ branching ratio by 40% and generate a B–type amplitude, ii) the vector meson exchange contributions that are in general model dependent [22, 23] but can be parameterized $K_L \to \pi^0\gamma\gamma$ by an effective vector coupling a_V [23] :

$$\begin{aligned} A &= \frac{G_8 M_K^2 \alpha_{em}}{\pi} a_V (3 - z + r_\pi^2) \ , \\ B &= -\frac{2 G_8 M_K^2 \alpha_{em}}{\pi} a_V \ , \end{aligned} \tag{12}$$

where G_8 is the octet weak coupling, determined from $K \to \pi\pi$. Thus the contribution to $K_L \to \pi^0 e^+ e^-$ is determined by the value of a_V. The agreement with experimental $K_L \to \pi^0\gamma\gamma$ rate and spectrum would demand $a_V \sim 0.9$ [20] .

It would be desirable to have a theoretical understanding of this value. Indeed we have related a_V with the $K_L \to \gamma(q_1, \epsilon_1)\gamma^*(q_2, \epsilon_2)$ slope [24]. The $K_L \to \gamma(q_1, \epsilon_1)\gamma^*(q_2, \epsilon_2)$ amplitude, $A_{\gamma\gamma^*}(q_2^2)$, can be approximated by a linear slope $A_{\gamma\gamma^*}(q_2^2) \approx A_{\gamma\gamma}^{exp} \cdot (1 + bx)$, where $A_{\gamma\gamma}^{exp}$ is the experimental amplitude $A(K_L \to \gamma\gamma)$ and $x = q_2^2/m_K^2$. The slope can be estimated [24], $b_{exp} = 0.81 \pm 0.18$. Theoretically the slope b is also generated by vector meson exchange contribution.

We can evaluate now a_V and the $K_L \to \gamma\gamma^*$ slope b in factorization (FM), i.e. writing a *current*$\times$ *current* structure

$$\mathcal{L}_{FM} \ = \ 4\, k_F\, G_8 \,\langle\, \lambda\, J_\mu\, J^\mu \,\rangle \ + \ h.c. \quad , \tag{13}$$

where $\lambda \equiv \frac{1}{2}(\lambda_6 - i\lambda_7)$ and the fudge factor $k_F \sim \mathcal{O}(1)$ has to be determined phenomenogically. A satisfactory understanding of the model would require $k_F \sim 0.2 - 0.3$, to match the perturbative result.

There are two ways to derive the FM weak lagrangian generated by resonance exchange (this corresponds to different ways to determine the conserved current J_μ) [24, 25] :

($\mathcal{A}$) To evaluate the strong action generated by resonance exchange, and then perform the factorization procedure in Eq. (13). By this way, since we apply the FM procedure once the vectors have already been integrated out the lagrangian is generated at the kaon mass scale.

($\mathcal{B}$) Otherwise, we can first write down the spin–1 strong and weak chiral lagrangian. The general effective weak $VP\gamma$ lagrangian contributing to both $\mathcal{O}(p^6)$ $K \to \pi\gamma\gamma$ and $K_L \to \gamma\gamma^*$ processes [24] writes as

$$\mathcal{L}_W(VP\gamma) \;=\; G_8\, F_\pi^2\, \varepsilon_{\mu\nu\alpha\beta} \sum_{i=1}^{5} \kappa_i \,\langle\, V^\mu\, T_i^{\nu\alpha\beta} \rangle \;, \tag{14}$$

where $T_i^{\nu\alpha\beta}$ $i = 1,..,5$ are all the possible relevant $P\gamma$ structures , the κ_i are the dimensionless coupling constants to be determined and $F_\pi \simeq 93\,\mathrm{MeV}$. The brackets in Eq. (14) stand for a trace in the flavour space. We apply factorization and thus we derive the weak resonance couplings κ_i . We then integrate out the resonance fields so that the effective lagrangian is generated at the scale of the resonance.

In principle the two effective actions do not coincide and phenomenology may prefer one pattern [25]. In the case at hand, $\mathcal{A}$ and $\mathcal{B}$ give different structures, however they both generate a good phenomenology with one free parameter k_F , i.e.

$$a_V \;\simeq\; -0.72 \qquad , \qquad b \;\simeq\; 0.8 - 0.9 \;, \tag{15}$$

but with different value of k_F : $\mathcal{A} \Rightarrow k_F = 1$ while $\mathcal{B} \Rightarrow k_F = 0.2$. Interestingly this seems to suggest that the matching should be performed at the resonance scale.

Very interestingly the new data from KTeV [18] confirms sharply our prediction for a_V : $a_V = -0.72 \pm 0.05 \pm 0.06$ and show a clear evidence of events at low z. This turns in a more stringent determination for the CP conserving contribution to $K_L \to \pi^0 e^+ e^-$: $1. < B(K_L \to \pi^0 e^+ e^-) \cdot 10^{12} < 4$ [12, 24].

4 Conclusions

We think that the recent experimental results in $K-$decays, for instance ε'/ε and $K_L \to \pi^0\gamma\gamma$, let us hope to make sensitive tests of the SM and of its possible extensions. From the theoretical side we expect an improvement in matching long and short distance contributions and this would lead to an accurate determination of low energy parameters, for instance a reliable relation between a_S and a_+ in (8).

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